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Discrete-Valued Vector Reconstruction by Optimization with Sum of Sparse Regularizers

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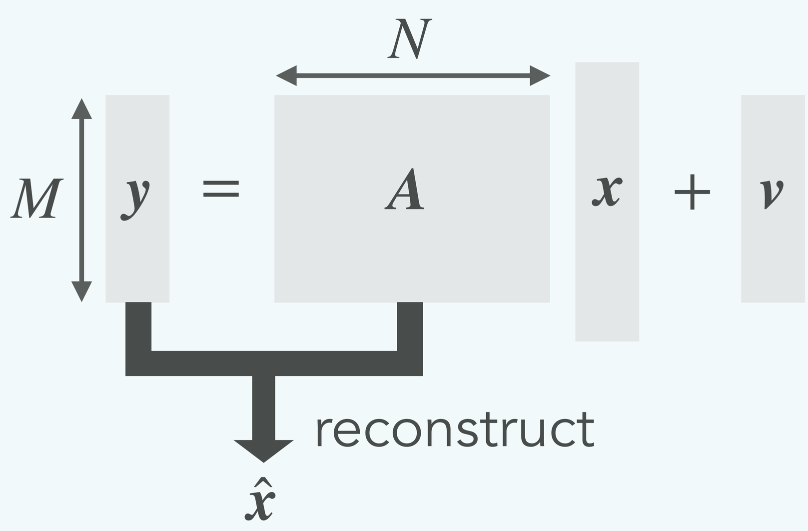
Abstract

We propose a **possibly nonconvex** optimization problem to **reconstruct a discrete-valued vector from its underdetermined linear measurements**. The proposed sum of sparse regularizers (SSR) optimization uses **the sum of sparse regularizers** as a regularizer for the discrete-valued vector. We also propose two proximal splitting algorithms for the SSR optimization problem on the basis of alternating direction method of multipliers (ADMM) and primal-dual splitting (PDS). The ADMM based algorithm can achieve faster convergence, whereas the PDS based algorithm does not require the computation of any inverse matrix.

1. Discrete-Valued Vector Reconstruction

Goal: Reconstruct a discrete-valued vector $\boldsymbol{x} \in \{r_1, \ldots, r_L\}^N \subset \mathbb{R}^N$ (distribution: $p_\ell = \Pr(x_n = r_\ell)$ $(\ell = 1, \ldots, L)$)
from underdetermined linear measurements $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{v} \in \mathbb{R}^M$ $(M < N)$



$\boldsymbol{A} \in \mathbb{R}^{M \times N}$: measurement matrix
$\boldsymbol{v} \in \mathbb{R}^M$: noise vector

Application
- ✦ signal detection for overloaded multiple-input multiple-output (MIMO)
- ✦ multiuser detection for machine to machine communications
- ✦ faster-than-Nyquist (FTN) signaling

2. Proposed Method

Sum of sparse regularizers (SSR) optimization

idea: $\boldsymbol{x} - r_\ell \boldsymbol{1}$ has some zero elements because $\boldsymbol{x} \in \{r_1, \ldots, r_L\}^N$

$$\underset{\boldsymbol{s} \in \mathbb{R}^N}{\text{minimize}} \sum_{\ell=1}^{L} q_\ell h\left(\boldsymbol{s} - r_\ell \boldsymbol{1}\right) + \frac{\lambda}{2}\|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{s}\|_2^2 \quad (q_\ell \geq 0, \lambda > 0 : \text{parameters})$$

h: sparse regularizer

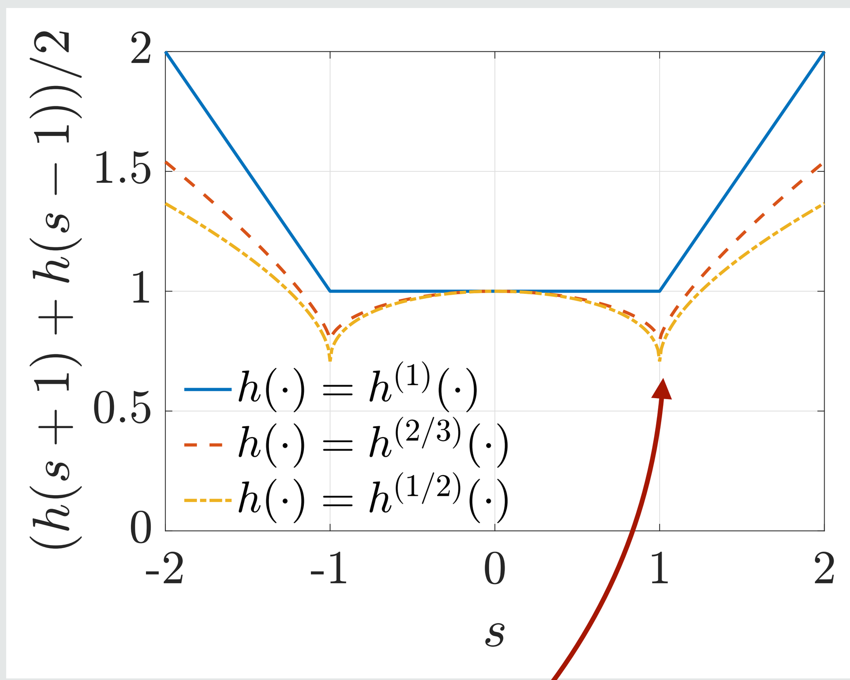
Example:
- ✦ convex
  - $h^{(1)}(\boldsymbol{u}) = \|\boldsymbol{u}\|_1$
- ✦ nonconvex
  - $h^{(p)}(\boldsymbol{u}) = \|\boldsymbol{u}\|_p^p$
  - $h^{(0)}(\boldsymbol{u}) = \|\boldsymbol{u}\|_0$
  - $h^{(1-2)}(\boldsymbol{u}) = \|\boldsymbol{u}\|_1 - \|\boldsymbol{u}\|_2$ [1]

ADMM-SSR: ADMM [2] based algorithm for SSR optimization
✓ faster convergence, require matrix inversion

PDS-SSR: PDS [3] based algorithm for SSR optimization
✓ slower convergence, no matrix inversion

Example: Binary vector reconstruction ($\boldsymbol{x} \in \{-1, 1\}^N$)
- ✦ $(r_1, r_2) = (-1, 1)$
- ✦ $(p_1, p_2) = (1/2, 1/2)$

⟶ regularizer: $\frac{1}{2}(h(s+1) + h(s-1))$



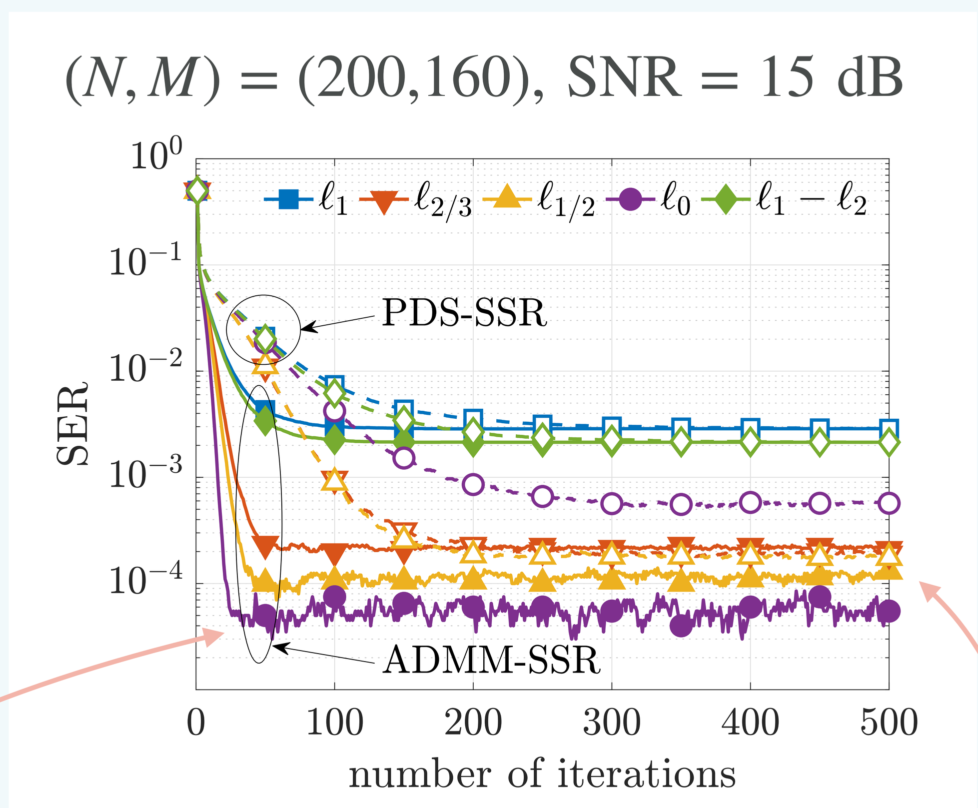
minimum value at 1 and −1

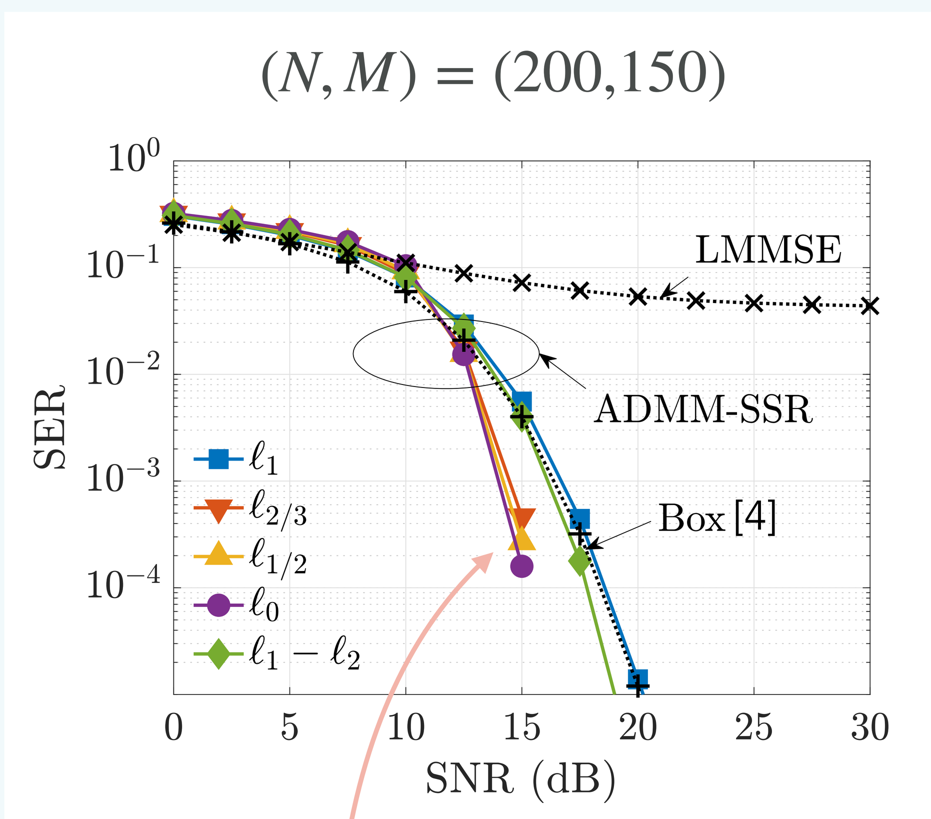
✓ The proposed approach can use any proximable sparse regularizer for $h(\cdot)$.
✓ The proposed approach can also be applied to the reconstruction of **complex** discrete-valued vector.

3. Simulation Results

Binary vector reconstruction ($\boldsymbol{x} \in \{-1, 1\}^N$)
- ✦ $(r_1, r_2) = (-1, 1)$
- ✦ $(p_1, p_2) = (1/2, 1/2)$
- ✦ $\boldsymbol{A}$: i.i.d. Gaussian
- ✦ sparse regularizer $h(\cdot)$
  - ℓ_1 norm
  - $\ell_{2/3}$ norm
  - $\ell_{1/2}$ norm
  - ℓ_0 norm
  - $\ell_1 - \ell_2$ difference [1]

($\ell_{2/3}$, $\ell_{1/2}$, ℓ_0, $\ell_1 - \ell_2$: nonconvex)





ADMM-SSR converges faster than PDS-SSR

The proposed algorithms with nonconvex regularizers, especially with the ℓ_p and ℓ_0 norms, can achieve much better SER performance.

[1] P. Yin et al., “Minimization of ℓ_{1-2} for compressed sensing,” SIAM J. Sci. Comput., vol. 37, no. 1, pp. A536–A563, Feb. 2015.
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[3] L. Condat, “A primal-dual splitting method for convex optimization involving Lipschitzian, proximable and linear composite terms,” J. Optim. Theory Appl., vol. 158, no. 2, pp. 460–479, Aug. 2013.
[4] P. H. Tan et al., “Constrained maximum-likelihood detection in CDMA,” IEEE Trans. Commun., vol. 49, no. 1, pp. 142–153, Jan. 2001.

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