Discreteness-Aware Decoding for Overloaded Non-Orthogonal STBCs via Convex Optimization

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***Abstract*—This paper proposes a decoding scheme for non-orthogonal space-time block codes (NO-STBCs) in overloaded multiple-input multiple-output (MIMO) systems, where the number of different transmitted streams is greater than that of receive antennas. For the underdetermined decoding problem, we iteratively solve a convex optimization problem with the update of the parameters in the objective function, which utilizes the discreteness of the transmitted symbols. For the NO-STBC based on cyclic division algebra, we can reduce the order of computational complexity of the algorithm by taking advantage of the structure of the code. Simulation results show that the proposed algorithm outperforms conventional schemes in the decoding of large-scale overloaded NO-STBCs.**

***Index Terms*—overloaded MIMO, non-orthogonal STBC, convex optimization**

I. Introduction

FOR multiple-input multiple-output (MIMO) communications, non-orthogonal space-time block codes (NO-STBCs) have been studied to achieve both high rate and high diversity order [1]. In [2], for example, a NO-STBC has been proposed by using cyclic division algebra (CDA), which can achieve both the full diversity and the information losslessness under maximum likelihood (ML) decoding. Moreover, the rate of the code is equal to the number of transmit antennas. Since ML decoding becomes infeasible as the number of antennas increases, several low-complexity schemes have been proposed, e.g., local neighborhood search [3], [4], belief propagation [5], and probabilistic data association [6].

Because of the limitation of size, physical weight, and/or power consumption at the receiver, sufficient number of receive antennas might be unavailable in practical systems, such as MIMO downlink communications with mobile terminals. MIMO systems are called overloaded (or underdetermined) when the number of different transmitted streams is greater than that of receive antennas [7]. The decoding problem of NO-STBCs becomes underdetermined in such overloaded MIMO systems when the code rate equals the number of transmit antennas, and thus the performance of the conventional decoding schemes for NO-STBCs significantly degrades for the overloaded scenarios. Since the size of the effective channel matrix in the decoding of NO-STBCs is considerably larger than that in uncoded MIMO systems, the decoding becomes a larger-scale problem even when the number of antennas is rather small. Hence, most signal detection schemes proposed for overloaded MIMO (e.g., [7], [8]) have a prohibitive computational complexity because they are partly based on ML decoding. The decoding of NO-STBCs in overloaded MIMO systems has not been discussed in the literature.

In this paper, we propose a decoding scheme for large-scale NO-STBCs in overloaded MIMO systems. The proposed scheme is based on iterative weighted sum-of-absolute-value (IW-SOAV) optimization [9], which has been originally proposed for signal detection in massive overloaded MIMO systems. It takes advantage of the discreteness of the transmitted symbols and iteratively solves a convex optimization problem with the update of the weights in the objective function. Note that the performance of IW-SOAV largely depends on the structure of the channel matrix, which are completely different in uncoded MIMO signal detection and the decoding of NO-STBCs. Hence, good performance of IW-SOAV is not necessarily trivial in the decoding of NO-STBCs. Since IW-SOAV generally works better for larger-scale problems, the decoding of NO-STBCs with the large-scale effective channel matrix can be one of remarkable applications of IW-SOAV. For the NO-STBC based on CDA, we also propose a method to reduce the order of the computational complexity of IW-SOAV by utilizing the structure of the code. Simulation results show that the proposed IW-SOAV outperforms conventional schemes in terms of bit error rate (BER) for overloaded MIMO systems with around or more than ten transmit antennas, and is also robust to spatially correlated channels.

The rest of the paper is organized as follows. We describe the system model in Section II and present the proposed decoding scheme in Section III. Section IV shows some simulation results and Section V gives the conclusion.

We use the following notations in this paper. We denote the real part and the imaginary part by $\mathrm{Re}\{\cdot\}$ and $\mathrm{Im}\{\cdot\}$, respectively. The transpose, the Hermitian transpose, the imaginary unit, an $N \times N$ identity matrix, and the vector whose elements are all 0 are indicated by $(\cdot)^{\mathsf{T}}$, $(\cdot)^{\mathsf{H}}$, j, $\boldsymbol{I}_N$, and $\boldsymbol{0}$, respectively. For a matrix $\boldsymbol{U} = [\boldsymbol{u}_1 \cdots \boldsymbol{u}_N] \in \mathbb{C}^{M \times N}$, $\mathrm{vec}(\boldsymbol{U})$ is given by $\mathrm{vec}(\boldsymbol{U}) = \left[\boldsymbol{u}_1^{\mathsf{T}} \cdots \boldsymbol{u}_N^{\mathsf{T}}\right]^{\mathsf{T}} \in \mathbb{C}^{MN}$. We denote the sign function by $\mathrm{sign}(\cdot)$ and the Kronecker product by $\otimes$.

II. System Model

In this paper, we consider MIMO communications with N_{t} transmit antennas and N_{r} receive antennas. By using a STBC,

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we send K complex data symbols $\tilde{s}_1, \ldots, \tilde{s}_K \in \mathbb{C}$ during T time slots. We define the STBC matrix as $\tilde{\boldsymbol{X}} = [\tilde{\boldsymbol{x}}_1 \cdots \tilde{\boldsymbol{x}}_T] \in \mathbb{C}^{N_t \times T}$, where $\tilde{\boldsymbol{x}}_t = [\tilde{x}_{1,t} \cdots \tilde{x}_{N_t,t}]^\mathsf{T} \in \mathbb{C}^{N_t}$ $(t = 1, \ldots, T)$ indicates the transmitted signal vector at the tth time slot and $\tilde{x}_{n_t,t}$ is the transmitted symbol from the n_tth transmit antenna $(n_t = 1, \ldots, N_t)$. In linear dispersion STBCs, the STBC matrix $\tilde{\boldsymbol{X}}$ is given by

$$\tilde{\boldsymbol{X}} = \sum_{k=1}^{K} \tilde{\boldsymbol{C}}_k \tilde{s}_k, \tag{1}$$

where $\tilde{\boldsymbol{C}}_k \in \mathbb{C}^{N_t \times T}$ is a weight matrix corresponding to the data symbol $\tilde{s}_k$. In [2], for example, the NO-STBC matrix

$$\tilde{\boldsymbol{X}} = \sum_{n_t=0}^{N_t-1} \begin{bmatrix} \bar{s}_{0,n_t} & \delta \bar{s}_{N_t-1,n_t}\omega_{N_t}^{n_t} & \cdots & \delta \bar{s}_{1,n_t}\omega_{N_t}^{(N_t-1)n_t} \\ \bar{s}_{1,n_t} & \bar{s}_{0,n_t}\omega_{N_t}^{n_t} & \cdots & \delta \bar{s}_{2,n_t}\omega_{N_t}^{(N_t-1)n_t} \\ \bar{s}_{2,n_t} & \bar{s}_{1,n_t}\omega_{N_t}^{n_t} & \cdots & \delta \bar{s}_{3,n_t}\omega_{N_t}^{(N_t-1)n_t} \\ \vdots & \vdots & \vdots & \vdots \\ \bar{s}_{N_t-2,n_t} & \bar{s}_{N_t-3,n_t}\omega_{N_t}^{n_t} & \cdots & \delta \bar{s}_{N_t-1,n_t}\omega_{N_t}^{(N_t-1)n_t} \\ \bar{s}_{N_t-1,n_t} & \bar{s}_{N_t-2,n_t}\omega_{N_t}^{n_t} & \cdots & \bar{s}_{0,n_t}\omega_{N_t}^{(N_t-1)n_t} \end{bmatrix} \rho^{n_t} \tag{2}$$

has been proposed by using CDA, where $\bar{s}_{n_t,n_t'} = \tilde{s}_{n_t N_t + n_t' + 1} \in \mathbb{C}$ $(n_t, n_t' = 0, \ldots, N_t - 1)$ are the complex data symbols to be sent, and $\omega_{N_t} = e^{j\frac{2\pi}{N_t}}$. Since we use $T = N_t$ time slots to send $K = N_t^2$ symbols in (2), the rate of this NO-STBC is $K/T = N_t$. Moreover, when $\delta = e^{\sqrt{5}j}$ and $\rho = e^j$, the full diversity is also achieved under ML decoding [2].

The received signal matrix $\tilde{\boldsymbol{Y}} \in \mathbb{C}^{N_r \times T}$ corresponding to $\tilde{\boldsymbol{X}}$ during T time slots is given by

$$\tilde{\boldsymbol{Y}} = \tilde{\boldsymbol{H}}\tilde{\boldsymbol{X}} + \tilde{\boldsymbol{V}}, \tag{3}$$

where $\tilde{\boldsymbol{H}} \in \mathbb{C}^{N_r \times N_t}$ is the channel matrix and $\tilde{\boldsymbol{V}} \in \mathbb{C}^{N_r \times T}$ is the zero mean additive white Gaussian noise matrix. From (1) and (3), we have $\tilde{\boldsymbol{Y}} = \sum_{k=1}^{K} \tilde{\boldsymbol{H}}\tilde{\boldsymbol{C}}_k \tilde{s}_k + \tilde{\boldsymbol{V}}$ and hence $\tilde{\boldsymbol{y}} := \mathrm{vec}(\tilde{\boldsymbol{Y}}) \in \mathbb{C}^{N_r T}$ can be written as

$$\tilde{\boldsymbol{y}} = \sum_{k=1}^{K} (\boldsymbol{I}_T \otimes \tilde{\boldsymbol{H}})\mathrm{vec}(\tilde{\boldsymbol{C}}_k)\tilde{s}_k + \mathrm{vec}(\tilde{\boldsymbol{V}}) \tag{4}$$

$$= (\boldsymbol{I}_T \otimes \tilde{\boldsymbol{H}})\tilde{\boldsymbol{C}}\tilde{\boldsymbol{s}} + \tilde{\boldsymbol{v}} \tag{5}$$

$$= \tilde{\boldsymbol{A}}\tilde{\boldsymbol{s}} + \tilde{\boldsymbol{v}}, \tag{6}$$

where $\tilde{\boldsymbol{s}} = [\tilde{s}_1 \cdots \tilde{s}_K]^\mathsf{T} \in \mathbb{C}^K$, $\tilde{\boldsymbol{v}} = \mathrm{vec}(\tilde{\boldsymbol{V}}) \in \mathbb{C}^{N_r T}$, $\tilde{\boldsymbol{C}} = \left[\mathrm{vec}(\tilde{\boldsymbol{C}}_1) \cdots \mathrm{vec}(\tilde{\boldsymbol{C}}_K)\right] \in \mathbb{C}^{N_t T \times K}$, and $\tilde{\boldsymbol{A}} = (\boldsymbol{I}_T \otimes \tilde{\boldsymbol{H}})\tilde{\boldsymbol{C}} \in \mathbb{C}^{N_r T \times K}$ [3]. Note that the size of the effective channel matrix $\tilde{\boldsymbol{A}} \in \mathbb{C}^{N_r T \times K}$ is much larger than that of $\tilde{\boldsymbol{H}} \in \mathbb{C}^{N_r \times N_t}$. We can rewrite the complex-valued signal model (6) as the real-valued signal model given by

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{s} + \boldsymbol{v}, \tag{7}$$

where $\boldsymbol{y} = \left[\mathrm{Re}\{\tilde{\boldsymbol{y}}\}^\mathsf{T}\ \mathrm{Im}\{\tilde{\boldsymbol{y}}\}^\mathsf{T}\right]^\mathsf{T} \in \mathbb{R}^{2N_r T}$, $\boldsymbol{s} = \left[\mathrm{Re}\{\tilde{\boldsymbol{s}}\}^\mathsf{T}\ \mathrm{Im}\{\tilde{\boldsymbol{s}}\}^\mathsf{T}\right]^\mathsf{T} \in \mathbb{R}^{2K}$, $\boldsymbol{v} = \left[\mathrm{Re}\{\tilde{\boldsymbol{v}}\}^\mathsf{T}\ \mathrm{Im}\{\tilde{\boldsymbol{v}}\}^\mathsf{T}\right]^\mathsf{T} \in \mathbb{R}^{2N_r T}$, and

$$\boldsymbol{A} = \begin{bmatrix} \mathrm{Re}\{\tilde{\boldsymbol{A}}\} & -\mathrm{Im}\{\tilde{\boldsymbol{A}}\} \\ \mathrm{Im}\{\tilde{\boldsymbol{A}}\} & \mathrm{Re}\{\tilde{\boldsymbol{A}}\} \end{bmatrix} \in \mathbb{R}^{2N_r T \times 2K}. \tag{8}$$

Algorithm 1 IW-SOAV for decoding STBCs

a) Set $\hat{\boldsymbol{s}} = \boldsymbol{0}$.
b) Iterate i)–iii) for L times.
   i) Update w_k^+ and w_k^-.
   ii) Set $\varepsilon \in (0, 1)$, $\gamma > 0$, $\boldsymbol{r}_0 \in \mathbb{R}^{2N_t^2}$, and $M_{\mathrm{itr}} \in \mathbb{N}$.
   iii) For $m = 0, 1, 2, \cdots, M_{\mathrm{itr}}$, iterate

$$\begin{cases} \boldsymbol{z}_m = \mathrm{prox}_{\gamma f_{\boldsymbol{w}}}(\boldsymbol{r}_m) \\ \theta_m \in [\varepsilon, 2 - \varepsilon] \\ \boldsymbol{r}_{m+1} = \boldsymbol{r}_m + \theta_m \Big(\big(\boldsymbol{I}_{2N_t^2} + \alpha\gamma \boldsymbol{A}^\mathsf{T}\boldsymbol{A}\big)^{-1} \\ \qquad \cdot \big(2\boldsymbol{z}_m - \boldsymbol{r}_m + \alpha\gamma\boldsymbol{A}^\mathsf{T}\boldsymbol{y}\big) - \boldsymbol{z}_m \Big) \end{cases}$$

   and let $\hat{\boldsymbol{s}} = \boldsymbol{z}_{M_{\mathrm{itr}}}$.
c) Compute $\mathrm{sign}(\hat{\boldsymbol{s}}) \in \{1, -1\}^{2N_t^2}$ as the estimate of $\boldsymbol{s}$.

When we use the NO-STBC given by (2) and assume $N_r < N_t$, the decoding is an underdetermined problem because $2K = 2N_t T > 2N_r T$ and hence $\boldsymbol{A}$ becomes a fat matrix.

III. Proposed Decoding Scheme

In this section, we present a decoding scheme to estimate the transmitted symbols $\boldsymbol{s}$ on the basis of IW-SOAV [9]. We use the NO-STBC given by (2) with $\delta = e^{\sqrt{5}j}$ and $\rho = e^j$ and hence $T = N_t$ and $K = N_t^2$. We also propose a method to reduce the order of the computational complexity of IW-SOAV.

A. Decoding via IW-SOAV

IW-SOAV has been proposed for a massive overloaded MIMO signal detection, and can be applied to the signal model (7) with STBCs. In this paper, we assume quadrature phase shift keying (QPSK) modulation with $\tilde{s}_k \in \{1+j, -1+j, -1-j, 1-j\}$, though IW-SOAV can be extended for any quadratic amplitude modulation (QAM) [9]. Under this assumption, each element of $\boldsymbol{s}$ takes only 1 or -1. By taking advantage of the discreteness, IW-SOAV iteratively solves the following convex weighted sum-of-absolute-value (W-SOAV) optimization problem

$$\hat{\boldsymbol{s}} = \underset{\boldsymbol{z} \in \mathbb{R}^{2N_t^2}}{\arg\min}\ f_{\boldsymbol{w}}(\boldsymbol{z}) + \frac{\alpha}{2}\|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{z}\|_2^2, \tag{9}$$

where $f_{\boldsymbol{w}}(\boldsymbol{z}) = \sum_{k=1}^{2N_t^2} \left(w_k^+|z_k - 1| + w_k^-|z_k + 1|\right)$, z_k is the kth element of $\boldsymbol{z}$, and α (> 0) is a parameter. As discussed in [9], the function $f_{\boldsymbol{w}}(\cdot)$ can be regarded as a regularizer for the discrete-valued vector in $\{1, -1\}^{2N_t^2}$. The weight parameters w_k^+ and w_k^- are set to $1/2$ at the first iteration, and iteratively updated by using the estimate $\hat{\boldsymbol{s}}$ obtained at the previous iteration. In [9], they are updated as $w_k^+ = e^{\hat{\Lambda}_k}/(1 + e^{\hat{\Lambda}_k})$ and $w_k^- = 1/(1 + e^{\hat{\Lambda}_k})$, respectively, where $\hat{\Lambda}_k$ is an approximated posterior log likelihood ratio (LLR) of s_k calculated with the previous estimate.

In Algorithm 1, we summarize the proposed decoding scheme via IW-SOAV using Douglas-Rachford algorithm [10] to solve (9). After updating the weights in b) i), we solve the W-SOAV optimization (9) in b) ii) and b) iii). $\mathrm{prox}_{\gamma f_{\boldsymbol{w}}}(\cdot)$ is the proximity operator [10] of the function $f_{\boldsymbol{w}}(\cdot)$ (See [9]).

B. Complexity Reduction for Matrix Inversion

The computational complexity of IW-SOAV might be dominated by the matrix inversion $\left(\boldsymbol{I}_{2N_t^2} + \alpha\gamma \boldsymbol{A}^{\mathsf{T}}\boldsymbol{A}\right)^{-1} \in \mathbb{R}^{2N_t^2 \times 2N_t^2}$, which requires the complexity of $\mathcal{O}\left(N_t^6\right)$ by the direct calculation. For the NO-STBC given by (2), however, we can reduce the complexity by utilizing the structure of the matrix $\boldsymbol{A} \in \mathbb{R}^{2N_tN_r \times 2N_t^2}$.

To reduce the complexity, we rewrite $\left(\boldsymbol{I}_{2N_t^2} + \alpha\gamma \boldsymbol{A}^{\mathsf{T}}\boldsymbol{A}\right)^{-1}$ by using the fact that $\tilde{\boldsymbol{C}} \in \mathbb{C}^{N_t^2 \times N_t^2}$ is a scaled unitary matrix [3], [5], which means $\tilde{\boldsymbol{C}}\tilde{\boldsymbol{C}}^{\mathsf{H}} = N_t \boldsymbol{I}_{N_t^2}$. From the matrix inversion lemma [11], we have

$$\begin{aligned}&\left(\boldsymbol{I}_{2N_t^2} + \alpha\gamma \boldsymbol{A}^{\mathsf{T}}\boldsymbol{A}\right)^{-1} \\ &= \boldsymbol{I}_{2N_t^2} - \alpha\gamma \boldsymbol{A}^{\mathsf{T}}\left(\boldsymbol{I}_{2N_tN_r} + \alpha\gamma \boldsymbol{A}\boldsymbol{A}^{\mathsf{T}}\right)^{-1}\boldsymbol{A}. \qquad (10)\end{aligned}$$

The matrix $\left(\boldsymbol{I}_{2N_tN_r} + \alpha\gamma \boldsymbol{A}\boldsymbol{A}^{\mathsf{T}}\right)^{-1}$ is written with the matrix $\tilde{\boldsymbol{B}} := \left(\boldsymbol{I}_{N_tN_r} + \alpha\gamma \tilde{\boldsymbol{A}}\tilde{\boldsymbol{A}}^{\mathsf{H}}\right)^{-1} \in \mathbb{C}^{N_tN_r \times N_tN_r}$ as

$$\left(\boldsymbol{I}_{2N_tN_r} + \alpha\gamma \boldsymbol{A}\boldsymbol{A}^{\mathsf{T}}\right)^{-1} = \begin{bmatrix}\mathrm{Re}\{\tilde{\boldsymbol{B}}\} & -\mathrm{Im}\{\tilde{\boldsymbol{B}}\} \\ \mathrm{Im}\{\tilde{\boldsymbol{B}}\} & \mathrm{Re}\{\tilde{\boldsymbol{B}}\}\end{bmatrix}. \qquad (11)$$

We thus further calculate $\tilde{\boldsymbol{B}}$ as

$$\begin{aligned}\tilde{\boldsymbol{B}} &= \left(\boldsymbol{I}_{N_tN_r} + \alpha\gamma\left(\boldsymbol{I}_{N_t} \otimes \tilde{\boldsymbol{H}}\right)\tilde{\boldsymbol{C}}\tilde{\boldsymbol{C}}^{\mathsf{H}}\left(\boldsymbol{I}_{N_t} \otimes \tilde{\boldsymbol{H}}\right)^{\mathsf{H}}\right)^{-1} && (12)\\ &= \left(\boldsymbol{I}_{N_tN_r} + \alpha\gamma N_t\left(\boldsymbol{I}_{N_t} \otimes \tilde{\boldsymbol{H}}\right)\left(\boldsymbol{I}_{N_t} \otimes \tilde{\boldsymbol{H}}^{\mathsf{H}}\right)\right)^{-1} && (13)\\ &= \left(\boldsymbol{I}_{N_t} \otimes \left(\boldsymbol{I}_{N_r} + \alpha\gamma N_t \tilde{\boldsymbol{H}}\tilde{\boldsymbol{H}}^{\mathsf{H}}\right)\right)^{-1} && (14)\\ &= \boldsymbol{I}_{N_t} \otimes \left(\boldsymbol{I}_{N_r} + \alpha\gamma N_t \tilde{\boldsymbol{H}}\tilde{\boldsymbol{H}}^{\mathsf{H}}\right)^{-1}. && (15)\end{aligned}$$

The computational complexity for $\left(\boldsymbol{I}_{N_r} + \alpha\gamma N_t \tilde{\boldsymbol{H}}\tilde{\boldsymbol{H}}^{\mathsf{H}}\right)^{-1}$ is only $\mathcal{O}\left(N_tN_r^2\right)$ because it is dominated by the calculation of $\tilde{\boldsymbol{H}}\tilde{\boldsymbol{H}}^{\mathsf{H}}$. Hence, we can obtain $\tilde{\boldsymbol{B}}$ and $\left(\boldsymbol{I}_{2N_tN_r} + \alpha\gamma \boldsymbol{A}\boldsymbol{A}^{\mathsf{T}}\right)^{-1}$ with the complexity $\mathcal{O}\left(N_t^2N_r^2\right)$. It should be noted that we need to calculate the inverse matrix only once in the algorithm. From (10), once the matrix $\left(\boldsymbol{I}_{2N_tN_r} + \alpha\gamma \boldsymbol{A}\boldsymbol{A}^{\mathsf{T}}\right)^{-1}$ is obtained, we can update $\boldsymbol{r}_m$ only with the addition of the vectors and the multiplication of the matrix and the vector, which requires $\mathcal{O}\left(N_t^3N_r\right)$. The update of w_k^+ and w_k^- in b) i) of Algorithm 1 can also be computed with $\mathcal{O}\left(N_t^3N_r\right)$ per iteration [9]. Hence, the overall computational complexity of the proposed algorithm is $\mathcal{O}\left(N_t^3N_r\right)$ when $N_t > N_r$. In TABLE I, we compare the computational complexity of IW-SOAV and some conventional schemes, i.e., the linear minimum mean-square-error (LMMSE), reactive tabu search (RTS) [4], enhanced RTS (ERTS) [12], and ML. The order of the complexity of the proposed IW-SOAV with the complexity reduction is much lower than the conventional schemes.

TABLE I
COMPUTATIONAL COMPLEXITY

LMMSE	$\mathcal{O}\left(N_t^3N_r^3\right)$
RTS [4]	$\mathcal{O}\left(N_t^5N_r\right)$
ERTS [12]	$\mathcal{O}\left(N_t^5N_r\right) + \mathcal{O}\left(N_{\mathrm{RTS}}N_t^3N_r\right)$ (N_{RTS}: number of RTS)
ML	$\mathcal{O}\left(2^{2N_t^2}N_t^3N_r\right)$
IW-SOAV (w/o complexity reduction)	$\mathcal{O}\left(N_t^6\right)$
IW-SOAV (w/ complexity reduction)	$\mathcal{O}\left(N_t^3N_r\right)$

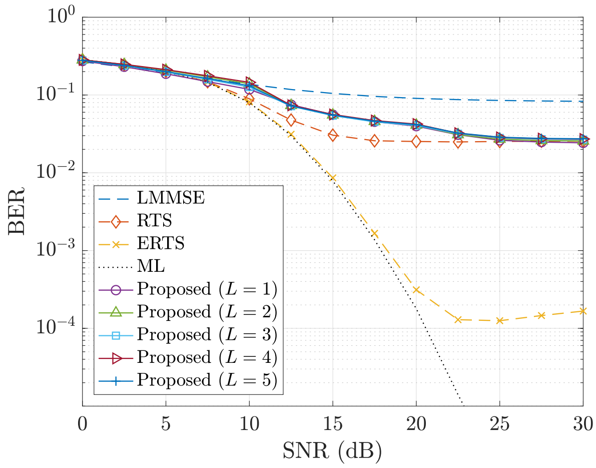

Fig. 1. BER performance in i.i.d. channels ($N_t = 3, N_r = 2$)

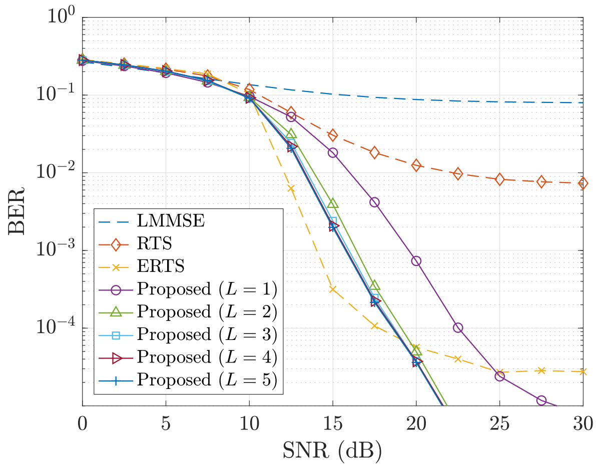

Fig. 2. BER performance in i.i.d. channels ($N_t = 9, N_r = 6$)

IV. SIMULATION RESULTS

We show some numerical results obtained by computer simulations. We use QPSK modulation and the NO-STBC given by (2) with $\delta = e^{\sqrt{5}j}$ and $\rho = e^{j}$. For the weight update in IW-SOAV, we use the same method as that in [9]. The parameters in IW-SOAV are $\varepsilon = 0.1$, $\gamma = 1$, $\boldsymbol{r}_0 = \boldsymbol{0}$, $M_{\mathrm{itr}} = 50$, and $\theta_m = 1.9$ $(m = 0, \ldots, M_{\mathrm{itr}})$.

Figures 1–3 show the BER performance of the proposed and conventional schemes for overloaded NO-STBCs. In the figures, we assume independent and identically distributed (i.i.d.) channels as $\tilde{\boldsymbol{H}} = \tilde{\boldsymbol{H}}_{\mathrm{i.i.d.}}$, where the elements of $\tilde{\boldsymbol{H}}_{\mathrm{i.i.d.}}$ are i.i.d. circular complex Gaussian variables with zero mean and unit variance. We denote the LMMSE decoding by "LMMSE", the RTS-based decoding [4] by "RTS", the ML decoding by "ML", and the proposed IW-SOAV scheme by "Proposed." We also plot the performance of ERTS [12], which has been proposed for signal detection in uncoded overloaded MIMO systems with tens of antennas. The parameters of RTS and ERTS are the same as those in [4] and [12], respectively. For the proposed IW-SOAV, the parameter α is the same as that in [9], i.e., $\alpha = 0.01, 0.1, 0.3,$ and 1 for SNR ranges of 0–10, 12.5–20, 22.5, and 25–30 [dB], respectively. In

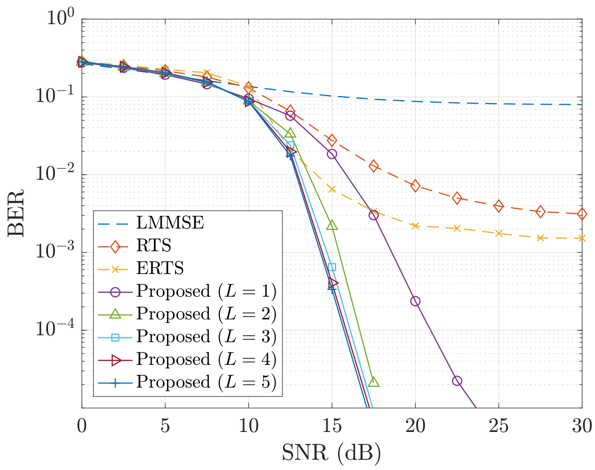

Fig. 3. BER performance in i.i.d. channels ($N_\mathrm{t} = 12, N_\mathrm{r} = 8$)

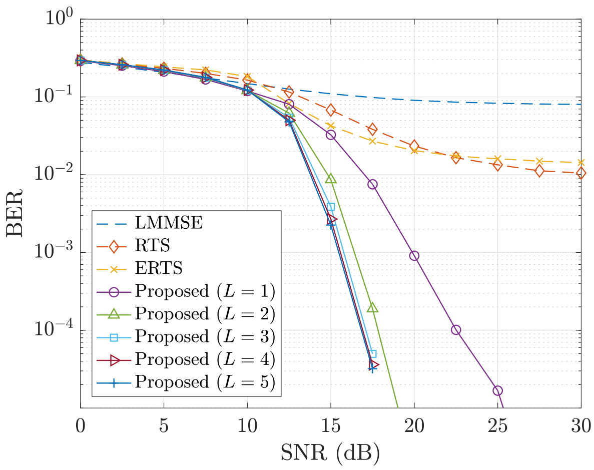

Fig. 4. BER performance in spatially correlated channels ($N_\mathrm{t} = 12, N_\mathrm{r} = 8$)

Fig. 1, where $(N_\mathrm{t}, N_\mathrm{r}) = (3, 2)$ with $\boldsymbol{A}$ of size 12×18, the proposed IW-SOAV has much worse performance than ML decoding for SNRs greater than 10 dB. Figure 2 shows the performance for more number of antennas $(N_\mathrm{t}, N_\mathrm{r}) = (9, 6)$, where the size of $\boldsymbol{A}$ is 108×162. In this case, we estimate the transmitted symbol vector in $\{1, -1\}^{162}$, which is equivalent to the signal detection for uncoded massive overloaded MIMO with $(N_\mathrm{t}, N_\mathrm{r}) = (81, 54)$. The ML decoding in this scenario is impractical because of the huge computational complexity, while the proposed IW-SOAV has better performance than other schemes for high SNRs. Although ERTS achieves better performance than the proposed IW-SOAV for SNRs around 15 dB, its complexity is about ten times larger than the proposed IW-SOAV with $L = 3$ [9]. In Fig. 3, where $(N_\mathrm{t}, N_\mathrm{r}) = (12, 8)$ with $\boldsymbol{A}$ of size 192×288, the proposed IW-SOAV outperforms the conventional schemes for all SNRs. A possible reason for the performance improvement is that the rows of $\boldsymbol{A}$ become more orthogonal when N_t and N_r increase.

Figure 4 shows the BER performance in spatially correlated channels. We assume $(N_\mathrm{t}, N_\mathrm{r}) = (12, 8)$ and $\tilde{\boldsymbol{H}} = \boldsymbol{\Phi}_\mathrm{r}^{\frac{1}{2}} \tilde{\boldsymbol{H}}_{\mathrm{i.i.d.}} \boldsymbol{\Phi}_\mathrm{t}^{\frac{1}{2}}$, where $\boldsymbol{\Phi}_\mathrm{r} \in \mathbb{C}^{N_\mathrm{r} \times N_\mathrm{r}}$ and $\boldsymbol{\Phi}_\mathrm{t} \in \mathbb{C}^{N_\mathrm{t} \times N_\mathrm{t}}$ are the correlation matrices at the receiver and transmitter, respectively [13]. We consider a linear array with equally spaced antennas and define $[\boldsymbol{\Phi}_\mathrm{r}]_{i_1, i_2} = J_0(|i_1 - i_2| \cdot 2\pi d_\mathrm{r}/\lambda)$ and $[\boldsymbol{\Phi}_\mathrm{t}]_{i_1, i_2} = J_0(|i_1 - i_2| \cdot 2\pi d_\mathrm{t}/\lambda)$, where $[\boldsymbol{\Phi}_\mathrm{r}]_{i_1, i_2}$ and $[\boldsymbol{\Phi}_\mathrm{t}]_{i_1, i_2}$ represent the (i_1, i_2) element of $\boldsymbol{\Phi}_\mathrm{r}$ and $\boldsymbol{\Phi}_\mathrm{t}$, respectively. Here, $J_0(\cdot)$ indicates the zeroth-order Bessel function of the first kind. We denote the antenna spacing at the receiver and the transmitter by d_r and d_t, respectively, and set $d_\mathrm{r} = d_\mathrm{t} = 0.5\lambda$ in the simulations, where λ is the wavelength. Figure 4 shows that the performance of the proposed IW-SOAV in spatially correlated channels is comparable to that in i.i.d. channels, while the BER of ERTS significantly degrades.

V. Conclusion

In this paper, we have proposed the decoding scheme for overloaded NO-STBCs and have evaluated its performance. The proposed scheme iteratively solves the W-SOAV optimization problem with the update of the weight parameters. We also propose the complexity reduction method for the proposed scheme with the NO-STBC based on CDA. Simulation results have shown that the proposed decoding scheme achieves better BER performance than the conventional schemes for large-scale problems. The results also suggest that IW-SOAV can achieve good performance by using the NO-STBC even in not-so-large overloaded MIMO systems. We thus conclude that the decoding of NO-STBCs can be one of remarkable applications of IW-SOAV. Future work includes theoretical performance analyses of the proposed decoding scheme.

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