Numerical Performance Evaluation of $\ell_1 - \ell_2$ Sparse Reconstruction Using Optical Analog Circuit

Soma Furusawa* , Kazunori Hayashi*, Kaito Kameda†, and Ryo Hayakawa†
* Graduate School of Informatics, Kyoto University, Kyoto, Japan, Tel/Fax: +81-75-753-9690/+81-75-753-9691
E-mail: furusawa.soma.82h@st.kyoto-u.ac.jp
E-mail: hayashi.kazunori.4w@kyoto-u.ac.jp
† Graduate School of Engineering Science, Osaka University, Osaka, Japan, Tel/Fax: +81-6-6850-6580
E-mail: kameda@sip.sys.es.osaka-u.ac.jp
E-mail: hayakawa.ryo.es@osaka-u.ac.jp

***Abstract*—Iterative shrinkage thresholding algorithm (ISTA), fast ISTA (FISTA) and alternating direction method of multipliers (ADMM) based algorithm are well known convex optimization algorithms based on proximal splitting for $\ell_1 - \ell_2$ optimization problem in compressed sensing. In our previous study, aiming at the implementation with optical analog devices, we have proposed constant inertial FISTA (CIFISTA) and an approximated ADMM based algorithm, which do not require the operation of division. In this paper, we consider to realize those algorithms with optical analog devices and show concrete configurations of analog circuits using optical devices. We have conducted computer experiments to evaluate the performance of each algorithm, taking into account the effects of the additive noise introduced by optical amplifiers assuming the implementation using optical devices for common optical fiber communications systems. From the numerical results, it has been confirmed that both the proposed CIFISTA and the approximated ADMM based algorithm can achieve faster convergence than ISTA, while the degradation of the steady state mean squared error (MSE) due to the circuit noise is much smaller than that of the conventional ADMM algorithm.**

I. Introduction

In order to achieve high-speed signal processing with low power consumption, the use of analog signal processing technology instead of conventional digital signal processing with electric devices has been attracting much attention [1]. Especially, optical analog device can compute the matrix-vector product by just passing optical signals through the device, and thus the implementation of machine learning techniques, such as deep learning, with the optical analog device has been investigated recently [2], [3], [4].

Compressed sensing is a method of reconstructing unknown sparse vector from underdetermined linear measurement which may include observation noise [5], [6], and various reconstruction algorithms have been proposed so far. One of well-known signal reconstruction approaches is the convex optimization method using proximal splitting [7]. Specifically, fast iterative shrinkage thresholding algorithm (FISTA) [8], which is the fast version of iterative shrinkage thresholding algorithm (ISTA) [9], is one of most famous algorithms of the approach. FISTA has a convergence rate of $O(k^{-2})$, which is known to be optimum for the first order method [10], [11], where k is the number of iterations in the algorithm. It is also known that alternating direction method of multipliers (ADMM) algorithm [12] can achieve comparable convergence rate to FISTA for $\ell_1 - \ell_2$ optimization problem.

In our previous study, we have considered to realize FISTA and the ADMM algorithm with optical analog devices. Since FISTA requires the operation of division by a dynamic value in the update equation of the inertial parameter, while it is difficult to calculate the division with optical analog device, we have proposed constant inertial FISTA (CIFISTA) [13] by simply setting the inertial parameter of FISTA to be a constant value. Moreover, we have developed an approximated ADMM based algorithm [14] by using different formulation from the existing ADMM algorithm for $\ell_1 - \ell_2$ optimization problem. Although both the conventional ADMM and the proposed approximated ADMM do not require any division operation in the update equations, and thus both algorithms could be implemented with optical devices, but the proposed algorithm has an advantage over the conventional ADMM that it is free from matrix inversion. In the previous studies [13], [14], we have numerically confirmed that both proposed algorithms can achieve comparable convergence performance to FISTA and the existing ADMM algorithm. However, the optical analog signal processing specific aspects, such as signal attenuation by optical devices or additive noise introduced by optical amplifiers, have not been taken into consideration.

In this paper, we evaluate the performance of the proposed CIFISTA and the proposed approximated ADMM algorithm to see the applicability of the proposed algorithms to the implementation with optical analog devices. In order to evaluate the power of the additive noise added at the optical amplifier, we present concrete optical circuit configuration of each algorithm and clarify the required amplifier power gain in the optical analog circuit. Since additive noise power at the amplifier depends on the amplifier power gain as well as the signal bandwidth, we can determine the noise power for each algorithm from the derived circuit configuration and the assumption of the signal bandwidth of 10 GHz, which corresponds to common optical fiber communications systems. From the numerical results, it can be observed that the CIFISTA, the approximated ADMM, and the conventional ADMM achieve comparable convergence rates, which is much faster than the rate of ISTA, even in the presence of the additive noise at the amplifier (circuit noise). On the other hand, when it comes to the degradation due to the circuit noise, ISTA achieves the lowest steady state mean squared error (MSE). However, the proposed CIFISTA and the proposed approximated ADMM still achieve acceptable steady state MSE, while the conventional ADMM suffers from larger degradation due to the circuit noise. After all, it can be concluded that both the proposed CIFISTA and the proposed approximated ADMM are suited for the implementation with optical analog circuits.

II. $\ell_1 - \ell_2$ Optimization Problem

In this paper, we consider the problem of estimating an unknown sparse vector $\boldsymbol{x} \in \mathbb{C}^N$ from the underdetermined linear measurement $\boldsymbol{y} \in \mathbb{C}^M$ obtained with a known sensing matrix $\boldsymbol{A} \in \mathbb{C}^{M \times N}$ as

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{n} \in \mathbb{C}^M \quad (M < N), \tag{1}$$

where $\boldsymbol{n} \in \mathbb{C}^M$ is a zero mean white observation noise vector. Such a problem is called compressed sensing [15], and in particular, the $\ell_1 - \ell_2$ optimization problem given by

$$\hat{\boldsymbol{x}}_{\ell_1-\ell_2} = \arg\min_{\boldsymbol{x}\in\mathbb{C}^N} \left(\lambda \|\boldsymbol{x}\|_1 + \frac{1}{2}\|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{y}\|_2^2 \right), \tag{2}$$

is often used to reconstruct the sparse signal. Here, $\lambda > 0$ is a regularization parameter that balances between ℓ_1 sparse regularization term of $\|\boldsymbol{x}\|_1$ and data fidelity term of $\frac{1}{2}\|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{y}\|_2^2$.

Several algorithms have been proposed for solving the $\ell_1-\ell_2$ optimization problem, but, in this paper, we focus on the convex optimization based algorithms using proximal splitting because they can achieve good reconstruction performance with low computational complexity in general.

III. Convex Optimization based Algorithms for $\ell_1 - \ell_2$ Optimization Problem

One of the simplest and most popular reconstruction algorithms will be ISTA shown in Algorithm 1. In the algorithm, $S_\delta(\cdot)$ is the proximity operator of ℓ_1 norm called as soft thresholding function and is defined by

$$S_\delta(x) := \begin{cases} (|x| - \delta)\dfrac{x}{|x|} & |x| \geq \delta \\ 0 & \text{otherwise} \end{cases}, \tag{3}$$

for $^\forall x \in \mathbb{C}$, $^\forall \delta > 0$ [16]. When the argument of $S_\delta(\cdot)$ is a vector, $S_\delta(\cdot)$ becomes a vector obtained by applying (3) for each element of the input vector. Since all operations involved in ISTA except for soft thresholding function* can be achieved with optical devices, ISTA could be implemented with optical analog devices, however, it suffers from slow rate of convergence.

*In this paper, we assume soft thresholding function is calculated in electrical domain by using efficient OEO (optical signal - electrical signal - optical signal) converter devices [17].

Algorithm 1 ISTA

Input: $\boldsymbol{x}[0]$, $\gamma > 0$.
Output: Estimate of $\boldsymbol{x}$
  for $k = 0, 1, 2, \ldots$ **do**
    $\boldsymbol{x}[k+1] = S_{\gamma\lambda}(\boldsymbol{x}[k] - \gamma \boldsymbol{A}^{\mathrm{H}}(\boldsymbol{A}\boldsymbol{x}[k] - \boldsymbol{y}))$
  end for
  return $\boldsymbol{x}[k+1]$

Algorithm 2 FISTA

Input: $\boldsymbol{x}[0]$, $\boldsymbol{z}[0]$, $t[0] > 0$, $\gamma > 0$.
Output: Estimate of $\boldsymbol{x}$
  for $k = 0, 1, 2, \ldots$ **do**
    $\boldsymbol{x}[k+1] = S_{\gamma\lambda}(\boldsymbol{z}[k] - \gamma \boldsymbol{A}^{\mathrm{H}}(\boldsymbol{A}\boldsymbol{z}[k] - \boldsymbol{y}))$
    $t[k+1] = \dfrac{1 + \sqrt{4t[k]^2 + 1}}{2}$
    $\boldsymbol{z}[k+1] = \boldsymbol{x}[k+1] + \dfrac{t[k] - 1}{t[k+1]}(\boldsymbol{x}[k+1] - \boldsymbol{x}[k])$
  end for
  return $\boldsymbol{x}[k+1]$

FISTA is a faster version of ISTA, where the acceleration is achieved by the employment of the update equation using not only the estimate in the previous iteration but also the estimate of one more previous iteration. Detailed steps of FISTA are shown in Algorithm 2. It is known that FISTA can achieve the optimum convergence rate among the first order methods [10], [11]. However, it is difficult to implement FISTA with optical analog devices, because it includes the operation of division with a dynamic value in the update equation of $\boldsymbol{z}[k+1]$.

In order to make FISTA realizable with optical analog devices, in our previous study [13], we have proposed CIFISTA given in Algorithm 3, in which $\frac{t[k]-1}{t[k+1]}$ is replaced by a constant inertial parameter c. In our preliminary numerical experiments, we have confirmed that CIFISTA can achieve comparable convergence performance to FISTA with a proper choice of the inertial parameter c.

ADMM is also a popular convex optimization algorithm using proximal splitting, and it is possible to solve the $\ell_1 - \ell_2$ optimization problem with ADMM as shown in Algorithm 4. ADMM could be implemented with optical analog devices because it does not require the division operation. However, one of major drawbacks of the conventional ADMM will be the necessity of the matrix inversion, which may restrict the

Algorithm 3 CIFISTA (proposed)

Input: $\boldsymbol{x}[0]$, $\boldsymbol{z}[0]$, $\gamma > 0$, $0 \leq c \leq 1$.
Output: Estimate of $\boldsymbol{x}$
  for $k = 0, 1, 2, \ldots$ **do**
    $\boldsymbol{x}[k+1] = S_{\gamma\lambda}(\boldsymbol{z}[k] - \gamma \boldsymbol{A}^{\mathrm{H}}(\boldsymbol{A}\boldsymbol{z}[k] - \boldsymbol{y}))$
    $\boldsymbol{z}[k+1] = \boldsymbol{x}[k+1] + c(\boldsymbol{x}[k+1] - \boldsymbol{x}[k])$
  end for
  return $\boldsymbol{x}[k+1]$

Algorithm 4 ADMM

Input: $\boldsymbol{z}[0]$, $\boldsymbol{v}[0]$, $\eta > 0$.
Output: Estimate of $\boldsymbol{x}$
  for $k = 0, 1, 2, \dots$ **do**
    $\boldsymbol{x}[k+1] = \left(\boldsymbol{A}^{\mathrm{H}}\boldsymbol{A} + \frac{1}{\eta}\boldsymbol{I}\right)^{-1} \cdot \left(\boldsymbol{A}^{\mathrm{H}}\boldsymbol{y} + \frac{1}{\eta}(\boldsymbol{z}[k] - \boldsymbol{v}[k])\right)$
    $\boldsymbol{z}[k+1] = S_{\eta\lambda}(\boldsymbol{x}[k+1] + \boldsymbol{v}[k])$
    $\boldsymbol{v}[k+1] = \boldsymbol{v}[k] + \boldsymbol{x}[k+1] - \boldsymbol{z}[k+1]$
  end for
  return $\boldsymbol{x}[k+1]$

Algorithm 5 Approximated ADMM (proposed)

Input: $\boldsymbol{x}[0]$, $\boldsymbol{z}[0]$, $\boldsymbol{v}[0]$, $\gamma > 0$, $\rho > 0$.
Output: Estimate of $\boldsymbol{x}$
  for $k = 0, 1, 2, \dots$ **do**
    $\boldsymbol{x}[k+1] = S_{\gamma\lambda/\rho}\left(\boldsymbol{x}[k] - \gamma\boldsymbol{A}^{\mathrm{H}}\left(\boldsymbol{A}\boldsymbol{x}[k] - \boldsymbol{z}[k] + \frac{1}{\rho}\boldsymbol{v}[k]\right)\right)$
    $\boldsymbol{z}[k+1] = \frac{1}{1+\rho}\left(\boldsymbol{y} + \rho\left(\boldsymbol{A}\boldsymbol{x}[k+1] + \frac{1}{\rho}\boldsymbol{v}[k]\right)\right)$
    $\boldsymbol{v}[k+1] = \boldsymbol{v}[k] + \rho(\boldsymbol{A}\boldsymbol{x}[k+1] - \boldsymbol{z}[k+1])$
  end for
  return $\boldsymbol{x}[k+1]$

application to large-scale problems[†].

In our previous study [14], we have derived a novel ADMM based algorithm obtained by using a different formulation from the existing ADMM. Update equations of the proposed algorithm are shown in Algorithm 5. With the new formulation, the $\ell_1 - \ell_2$ optimization problem appears in one of update equations of ADMM, therefore, we have approximated the update equation with a single iteration of ISTA in order to avoid the double loop algorithm. This is the reason why the algorithm is called "approximated" ADMM. Note that the proposed approximated ADMM does not include matrix inversion, and also that the approximated ADMM can achieve comparable convergence performance to the conventional ADMM in our preliminary numerical experiments.

IV. Proposed Optical Analog Circuit Configurations of $\ell_1 - \ell_2$ Optimization Algorithms

Since all reconstruction algorithms discussed in the previous section require iterative signal processing, it would be a natural approach to implement each algorithm with optical analog circuit, where one iteration of the algorithm corresponds to the optical signal propagation of one round in the circuit. As for the optical devices to implement the algorithms, we assume following optical devices are available: signal splitter (SS), adder, subtractor, matrix-vector multiplier, attenuator, amplifier, and delay element. Among the devices, we focus on the impact of SS, adder, and subtractor, because they are dominant devices in the circuit and are realized with a common optical device, namely the beam splitter (BS), which introduces large signal attenuation.

[†]Note that we do not assume that the matrix inversion is calculated with optical analog devices.

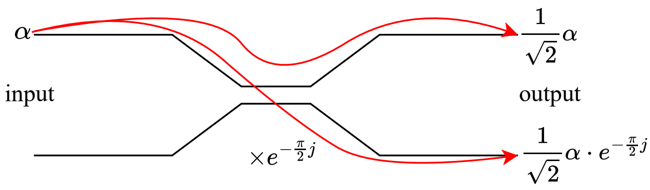

Fig. 1. Beam splitter (BS)

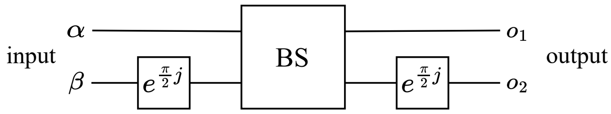

Fig. 2. Optical adder / subtractor

BS has two input ports and two output ports as depicted in Fig. 1. If we input signal $\alpha \in \mathbb{C}$ from one of the ports, then the output signal from the output port of the same side as the input is $\frac{1}{\sqrt{2}}\alpha$, while the output signal from the other side port is $\frac{1}{\sqrt{2}}\alpha \cdot e^{-\frac{\pi}{2}j}$, where j is an imaginary unit. Thus, if we give the phase shift of $e^{\frac{\pi}{2}j}$ to the lower port in Fig. 1, then BS can be used to realize SS, while the signal power of each output becomes half of the input signal power.

Optical adder and subtractor also can be realized by using BS and phase shifter. Fig. 2 shows the configuration of the adder and subtractor. Based on the property of BS, if we input signals ${}^{\forall}\alpha, \beta \in \mathbb{C}$ from the two input ports, then output signals from the two output ports will be

$$o_1 = \frac{1}{\sqrt{2}}\alpha + \frac{1}{\sqrt{2}}\beta \cdot e^{\frac{\pi}{2}j} \cdot e^{-\frac{\pi}{2}j} = \frac{1}{\sqrt{2}}(\alpha + \beta), \tag{4}$$

$$o_2 = \frac{1}{\sqrt{2}}\alpha \cdot e^{-\frac{\pi}{2}j} \cdot e^{\frac{\pi}{2}j} + \frac{1}{\sqrt{2}}\beta \cdot e^{\frac{\pi}{2}j} \cdot e^{\frac{\pi}{2}j} = \frac{1}{\sqrt{2}}(\alpha - \beta). \tag{5}$$

Thus, the addition and the subtraction can be realized with the optical analog devices in Fig. 2, while the power of the output signal becomes half of the desired signal as in the case of SS.

Optical analog circuits for the signal reconstruction have several SSs, adders and subtractors, therefore, the optical signal in the circuit suffers from signal attenuation during the propagation. However, at the input of soft thresholding function, which is required in all algorithms considered in this paper, the signal power must be an appropriate value because the soft thresholding function is a nonlinear function. Therefore, we insert optical amplifiers in the circuits before the soft thresholding function in order to compensate the signal attenuation due to BSs. Note that the number of required BSs in the optical circuit depends on the algorithms, and thus the required power gain of the amplifier also depends on the algorithms. Moreover, some optical attenuators must be

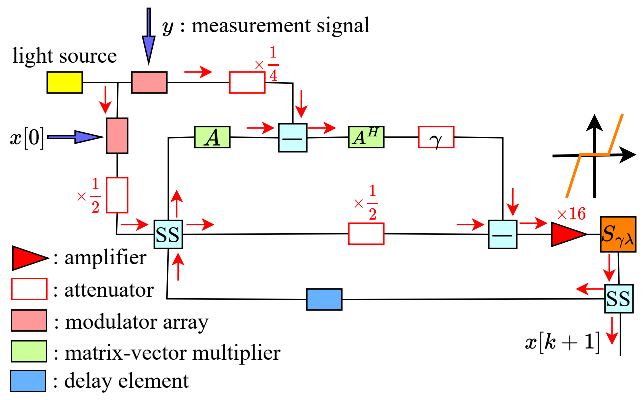

Fig. 3. Optical analog circuit configuration of ISTA

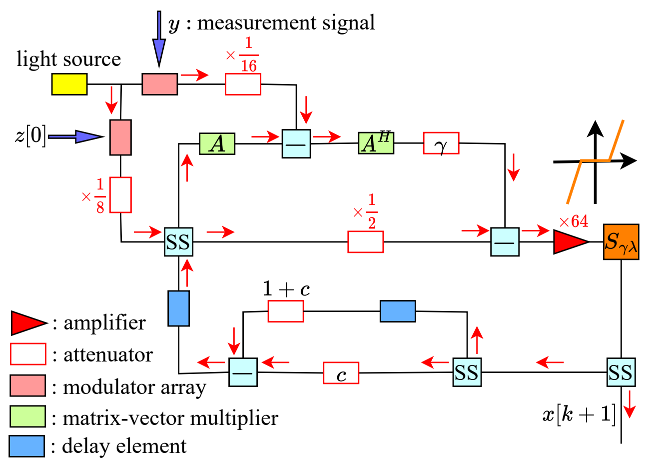

Fig. 4. Optical analog circuit configuration of CIFISTA

inserted at appropriate locations in the circuit to balance the signal power of each term before the addition or subtraction operations. Based on these considerations, we have derived the optical analog circuit configurations of ISTA, CIFISTA, conventional ADMM, and proposed approximated ADMM in Figs. 3 to 6, respectively.

V. Estimate of Additive Noise Power at Optical Amplifier

In the previous section, we have shown that optical amplifiers are necessary in the optical analog circuits of the $\ell_1 - \ell_2$ optimization algorithms. Generally speaking, additive noise is inevitable in the amplification of the signals. Therefore, the additive noise at the amplifier could be one of major performance limiting factors of the proposed approach. In this section, we estimate the power of the additive noise at the amplifier based on the noise model of optical amplifier.

According to [18], the power spectral density of amplified spontaneous emission (ASE) noise for a single erbium doped fiber amplifier (EDFA), which is one of most popular optical amplifiers, is given by

$$G_{\text{ASE}} = F(G-1)h\mu, \tag{6}$$

where F is the amplifier noise figure (NF), G is the amplifier power gain, h is Planck's constant, and μ is the frequency. Thus, the noise power is given by the product of the power spectral density G_{ASE} and the signal bandwidth.

In order to estimate the noise power, we have assumed to use optical devices for common optical fiber communications systems. Specifically, if we assume the wavelength of $1550\,\text{nm}$ corresponding to the standard single-mode fiber, then the frequency will be

$$\mu = \frac{3.0\times 10^8}{1550\times 10^{-9}} \approx 1.94\times 10^{14}. \tag{7}$$

Moreover, by using signal bandwidth of $10\,\text{GHz}$, which is typical for optical fiber communications, the ASE noise power added the optical amplifier with the power gain of G is given by

$$G_{\text{ASE}} \times 10\,\text{GHz} = (G-1)\cdot 2.56\times 10^{-9}, \tag{8}$$

where the ideal value of $F = 2$ is employed for NF. Some specific values of the additive noise power at the optical amplifier with different power gain G are summarized in Table I.

TABLE I
Estimation of Additive Noise Power for Different Amplifier Power Gain

amplifier power gain (G)	additive noise power
8	1.79×10^{-8}
16	3.84×10^{-8}
32	7.94×10^{-8}
64	1.61×10^{-7}
128	3.25×10^{-7}
256	6.53×10^{-7}

VI. Numerical Results

Here, we evaluate the performance of ISTA, CIFISTA, the conventional ADMM, and the proposed approximated ADMM in the presence of additive noise at the optical amplifier (circuit noise), which is assumed to be white Gaussian noise, via computer experiments.

In the experiments, we set the dimension of unknown sparse vector $\boldsymbol{x}$ to be $N = 500$ and that of linear measurement vector be $M = 200$. We set the number of non-zero elements of $\boldsymbol{x}$ as $K = 25$, and the non-zero elements are randomly generated with independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian (CSCG) distribution with zero-mean and variance $\sigma_{\boldsymbol{x}}^2 = 1$. Note that typical signal power in the optical fiber transmission is around 0.001, thus we have used 1,000 times larger values for the additive noise power at the optical amplifier than the values in Table I. The measurement matrix $\boldsymbol{A} \in \mathbb{C}^{M\times N}$ is generated by randomly choosing M rows of N-point unitary discrete Fourier transform (DFT) matrix. The measurement noise $\boldsymbol{n} \in \mathbb{C}^{M}$

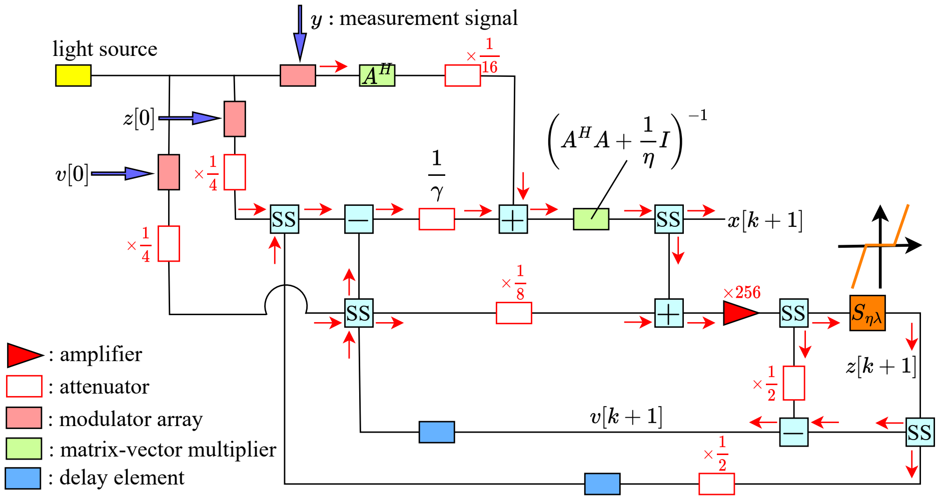

Fig. 5. Optical analog circuit configuration of conventional ADMM

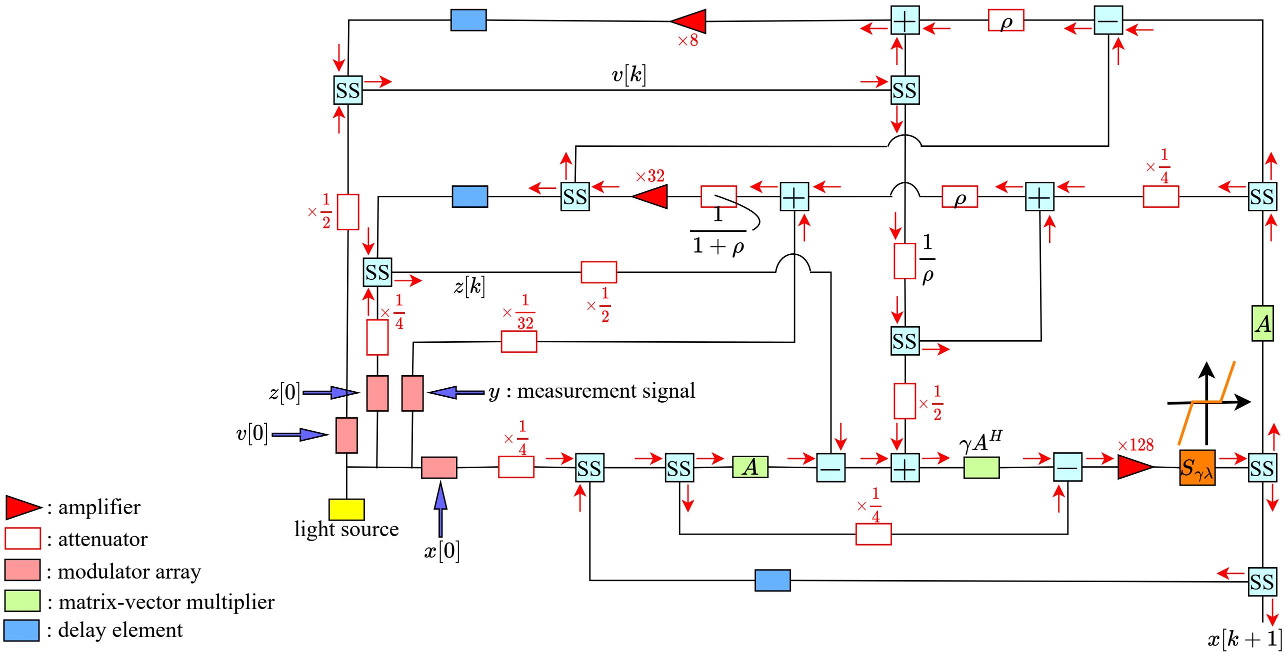

Fig. 6. Optical analog circuit configuration of approximated ADMM

is generated from zero-mean i.i.d. CSCG distribution with the variance corresponding to the measurement signal-to-noise ratio (SNR) of 30dB. We set the regularization parameter for the $\ell_1 - \ell_2$ optimization problem to be $\lambda = 0.02$, the constant inertial parameter of CIFISTA $c = 0.65$, and other parameters $\gamma = 0.99$, $\eta = 4.5 \times 0.99$, and $\rho = 0.20$. The initial values of the vectors in the algorithms are all set to $\boldsymbol{x}[0] = \boldsymbol{z}[0] = \boldsymbol{v}[0] = \mathbf{0}$. Under these settings, the MSE performance of each algorithm is evaluated by averaging the results obtained with 100 trials.

Fig. 7 shows the MSE performance of ISTA, CIFISTA, the conventional ADMM, and the proposed approximated ADMM in the absence of the circuit noise. In this case, the performance of FISTA is also shown in the same figure for comparison purpose. From the figure, we can see that all algorithms achieve almost the same steady state MSE, while the convergence rate of ISTA is much slower than other four algorithms. CIFISTA achieves similar convergence rate to FISTA in spite of the simplified algorithm with a constant inertial parameter. Also, the proposed approximated ADMM can achieve comparable convergence rate to the conventional ADMM without using matrix inversion.

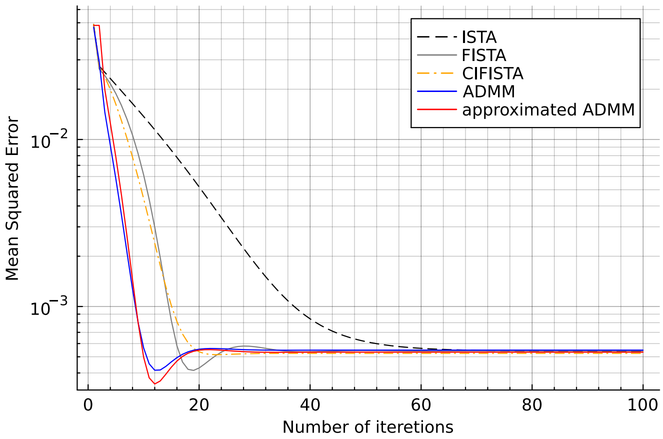

Fig. 7. Comparison of convergence characteristics (no circuit noise)

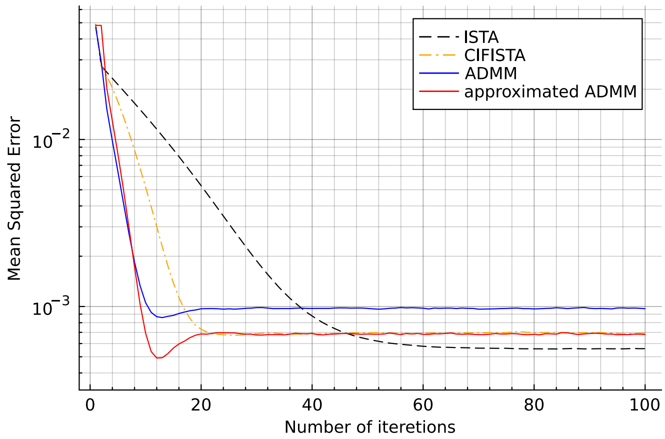

Fig. 8. Comparison of convergence characteristics (with circuit noise)

Fig. 8 shows the MSE performance of ISTA, CIFISTA, the conventional ADMM, and the proposed approximated ADMM in the presence of the circuit noise. As you can see from the figures, the degradation of the steady state MSE of ISTA is almost negligible. On the other hand, a certain degradation of the steady state MSE can be observed for CIFISTA and the approximated ADMM, while the conventional ADMM suffers from larger degradation of the steady state MSE. This could be mainly because the power gain of the optical amplifier is the smallest for the case with ISTA and is the largest for the case with the conventional ADMM. Moreover, the soft thresholding function also could have a certain impact on the steady state MSE performance, since it has a function to eliminate small noises, and only in the case of the conventional ADMM, half of the optical amplifier output with the largest power gain does not pass through the soft thresholding function.

Table II summarizes the comparison results of the convergence performance obtained by numerical experiments and the complexity of the optical analog circuits for the four algorithms. Judging from the balance between the steady state MSE and the convergence rate, both CIFISTA and the proposed approximated ADMM could be good choices for the implementation with optical analog circuits.

VII. Conclusions

In this paper, we have evaluated the performance of convex optimization based algorithms using proximal splitting for the $\ell_1 - \ell_2$ optimization problem assuming the implementation with optical analog devices. We have presented concrete configurations of optical analog circuits to realize ISTA, CIFISTA, the conventional ADMM, and the approximated ADMM. Moreover, we have estimated the additive noise power at the optical amplifier for each optimization algorithm assuming the implementation using optical devices for common optical fiber communications systems. From the numerical results obtained in the presence of the circuit noise, it can be concluded that both the proposed CIFISTA and the proposed approximated ADMM are more suited for the implementation with optical analog circuits than ISTA or the conventional ADMM.

Future work includes the investigation of the attenuation caused by other optical devices, and the implementation of soft thresholding function without using OEO converter.

Acknowledgment

This work has been supported in part by JST CREST JPMJCR21C3.

References

[1] N. Shlezinger, G. C. Alexandropoulos, M. F. Imani, Y. C. Eldar and D. R. Smith, “Dynamic Metasurface Antennas for 6G Extreme Massive MIMO Communications,” in *IEEE Wireless Communications*, vol. 28, no. 2, pp. 106-113, April 2021.
[2] Wetzstein, G., et al, “Inference in artificial intelligence with deep optics and photonics,” *Nature* 588, 39-47 2020.
[3] C. Huang, et al, “Prospects and applications of photonic neural networks,” *Advances in Physics: X*, 7:1, 2022.
[4] Zhou, H., et al, “Photonic matrix multiplication lights up photonic accelerator and beyond.” *Light Sci Appl* 11, 30 2022.
[5] M. Elad “Sparse and Redundant Representations,” *Springer*, 2010.
[6] I. Rish and G. Grabarnik, “Sparse Modeling: Theory, Algorithms, and Applications,” *CRC Press*, 2014.
[7] M. Nagahara, “Sparsity methods for systems and control,” *now Publishers*, 2020.
[8] A. Beck and M. Teboulle, “A Fast Iterative Shrinkage-Thresholding Algorithm for Linear Inverse Problems,” *SIAM Journal on Imaging Sciences*, vol. 2, no. 1, pp. 188-202, 2009.
[9] I. Daubechies, M. Defrise, and C. D. Mol, “An iterative thresholding algorithm for linear inverse problems with a sparsity constraint,” *Communications on Pure and Applied Mathematics*, vol. 57, no. 11, pp. 1413-1457, 2004.
[10] Y. Nesterov, *Introductory lectures on convex optimization: A basic course*, Springer, 2004.
[11] Y. Nesterov, “Gradient methods for minimizing composite functions,” *Mathematical Programming*, Vol. 140, No. 1, pp. 125-161, 2013.
[12] Eckstein, J., Bertsekas, D.P., “On the Douglas-Rachford splitting method and the proximal point algorithm for maximal monotone operators,” *Mathematical Programming* 55, 293-318 1992.

TABLE II

CHARACTERISTIC COMPARISON OF ALGORITHMS AS AN OPTICAL ANALOG CIRCUIT

	ISTA	CIFISTA	ADMM	approximated ADMM
Convergence Rate	slow	fast	fast	fast
Degradation due to Circuit Noise	very small	small	large	small
Circuit Complexity	simple	simple	complex	complex

[13] K. Kameda, R. Hayakawa, K. Hayashi and Y. Iiguni, "Performance Evaluation of FISTA with Constant Inertial Parameter," *2022 Asia-Pacific Signal and Information Processing Association Annual Summit and Conference (APSIPA ASC)*, Chiang Mai, Thailand, 2022, pp. 1713-1719.

[14] R. Lin and K. Hayashi, "An Approximated ADMM based Algorithm for $\ell_1 - \ell_2$ Optimization Problem," *2022 Asia-Pacific Signal and Information Processing Association Annual Summit and Conference (APSIPA ASC)*, Chiang Mai, Thailand, 2022, pp. 1720-1724.

[15] D. L. Donoho, "Compressed sensing," in *IEEE Transactions on Information Theory*, vol. 52, no. 4, pp. 1289-1306, April 2006.

[16] M. V. W. Zibetti, E. S. Helou, R. R. Regatte, and G. T. Herman, "Monotone FISTA With Variable Acceleration for Compressed Sensing Magnetic Resonance Imaging," in *IEEE Transactions on Computational Imaging*, vol. 5, no. 1, pp. 109-119, March 2019.

[17] K. Nozaki, S. Matsuo, T. Fujii, et al. , "Femtofarad optoelectronic integration demonstrating energy-saving signal conversion and nonlinear functions," *Nat. Photonics* Vol. 13, pp. 454-459, 2019.

[18] P. Poggiolini, G. Bosco, A. Carena, V. Curri, Y. Jiang, and F. Forghieri, "A Detailed Analytical Derivation of the GN Model of Non-Linear Interference in Coherent Optical Transmission Systems," *arXiv*: 1209.0394, 2014.